

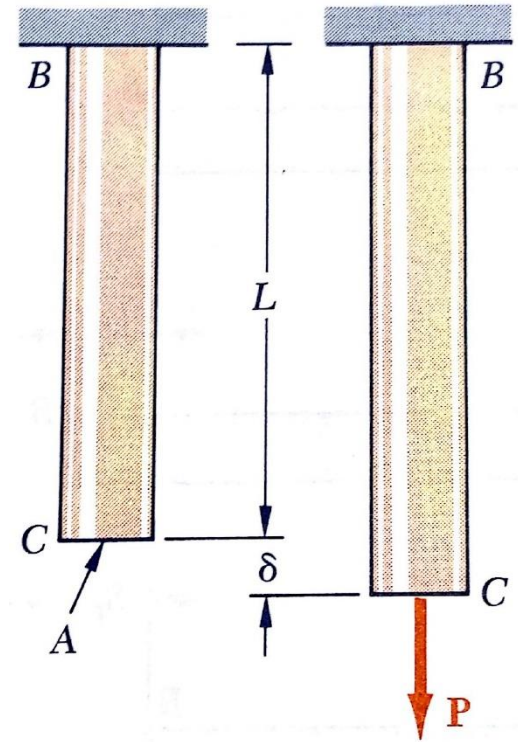
# Elasticity exercises

A rod of length  $L=500$  mm and cross-sectional area  $A=60$  mm<sup>2</sup> is made of an elastoplastic material having a modulus of elasticity  $E= 200$  GPa in its elastic range and a yield point  $\sigma_Y= 300$  MPa. The rod is subjected to an axial load until it is stretched 7 mm and the load is removed. What is the resulting permanent set?

A nylon thread is to be subjected to an 11-N tension. What is the required diameter of the thread if its modulus is 3.1 GPa, and the length of the thread should not increase by more than 1%.

An aluminum test specimen that is in the shape of a bar of length 300mm\*60mm\*15mm is subjected to two equal and opposite centric axial forces of magnitude P. What is the maximum allowable value of P and the corresponding total elongation of the specimen if  $E = 70 \text{ GPa}$  and  $\sigma_{allowable} = 200 \text{ MPa}$ .

The cylindrical rod AB has a length  $L = 2\text{m}$  and a 32mm diameter. It is made of a mild steel which is elastoplastic with  $E = 200\text{ GPa}$  and  $\sigma_y = 250\text{ MPa}$ . A force  $P$  is applied to the rod until its end A has moved down relative to its top side that is connected to a support. What is the maximum value of the force  $P$  and the permanent set of the rod after the force has been removed if the elongation is a) 3mm, b) 6mm?



A steel rod of length  $L$  and uniform cross section of area  $A$  is attached to rigid supports and is unstressed at a temperature of  $20\text{ C}$ . The steel is assumed to be elastoplastic with  $E = 200\text{ GPa}$  and  $\sigma_Y = 250\text{ MPa}$ . What is the stress in the rod after the temperature has been raised to  $150\text{ C}$  if the coefficient of thermal expansion is  $11.7 \cdot 10^{-6}/\text{C}$ ?

A cylindrical block of brass, that is 160 mm high and 120 mm in diameter is lowered into the ocean to a depth of 7500 m where the pressure is 75 MPa. Determine a) the change in height of the block, b) the change in its diameter, c) the change in its volume if the  $E = 105$  GPa and  $\nu = 0.35$ . What is the pressure which should be applied d) to its top and bottom faces only, and e) to its cylindrical surface only to cause the same change of volume as the hydrostatic pressure?

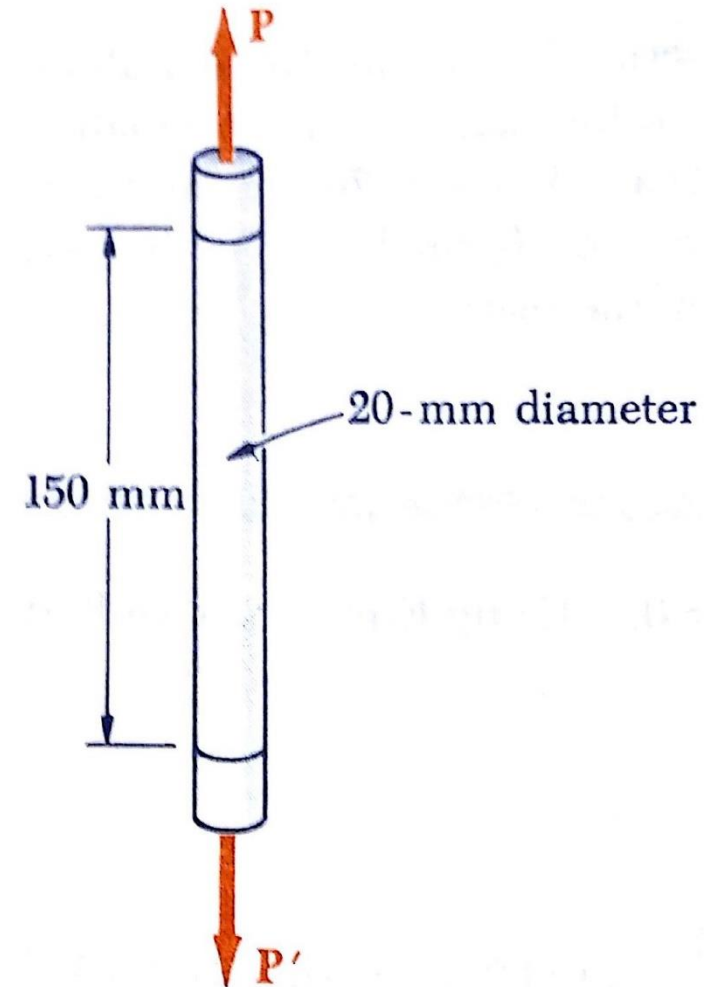
$$\sigma_{hyd} = \kappa \varepsilon \quad \text{or} \quad \sigma_{hyd} = \kappa \frac{\Delta V}{V} \qquad \kappa = \frac{E}{3(1-2\nu)} = \frac{2G(1+\nu)}{3(1-2\nu)}$$

What is the change in volume of a steel block of dimensions 80\*60\*40 mm when it is subjected to the hydrostatic pressure  $p= 180$  MPa? What are the changes in the length of each side?  $E= 200$  GPa,  $\nu= 0.29$ .

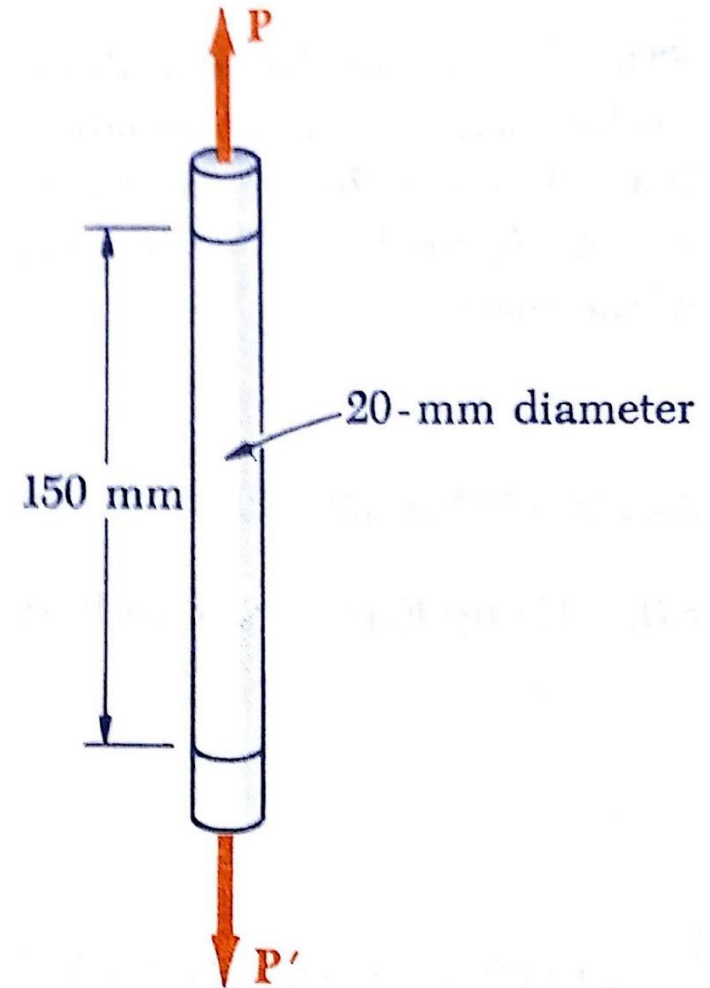
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In a standard tensile test an aluminum rod of 20 mm diameter is subjected to a tensile force of magnitude  $P = 30$  kN. What is the a) elongation of the rod in a 150 mm gage length and b) change in diameter of the rod if  $E = 70$  GPa and  $\nu = 0.35$ . Also consider c) the dilatation (the change in volume) of the rod.

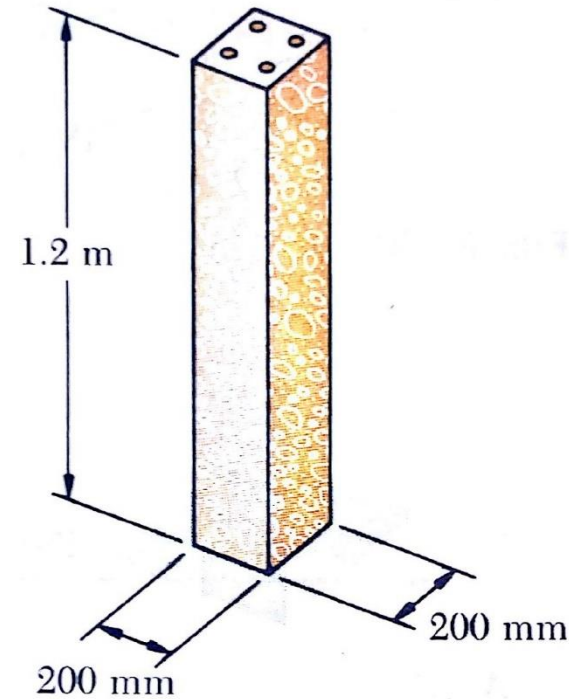


In a standard tensile test a 20 mm diameter rod made of an experimental polymer is subjected to a tensile force of magnitude  $P= 6$  kN. What are the modulus of elasticity, the modulus of rigidity and the poisson's ratio of the material if the elongation is 14 mm and decrease in diameter is 0.85 mm?

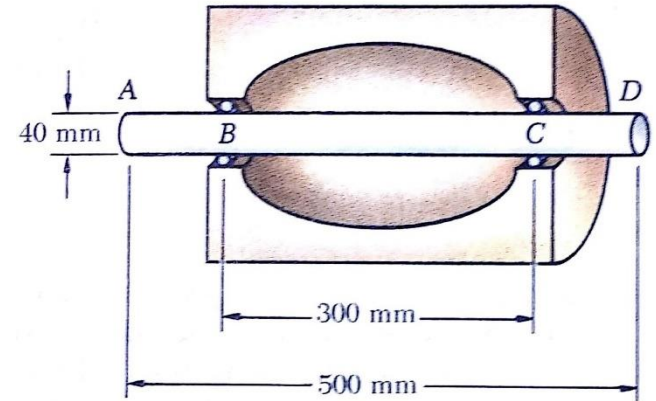


A 1.2 m concrete post is reinforced by four steel bars, each of 19 mm diameter. What are the normal stresses in the steel and concrete when a 700 kN centric force is applied to the post if the moduli for steel and concrete are  $E_s = 200 \text{ GPa}$ ,  $E_c = 25 \text{ GPa}$  ?

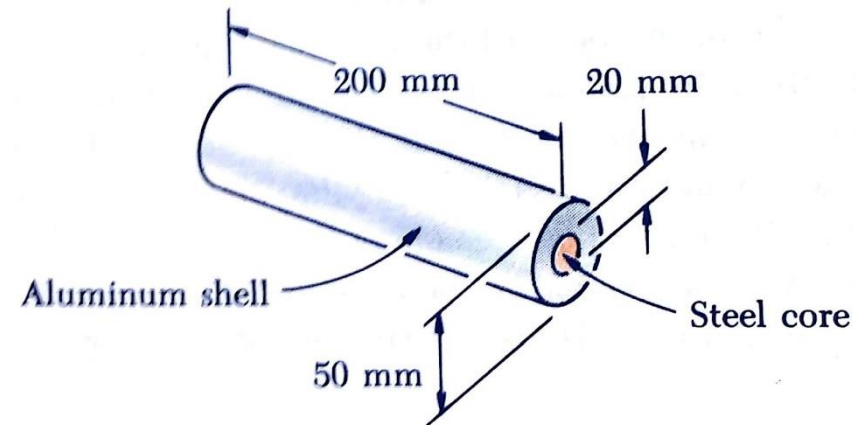
What are the normal stresses induced in the steel and in the concrete by a temperature rise of 45 C if the coefficients of thermal expansion are  $\alpha_s = 11.7 \cdot 10^{-6}/\text{C}$  and  $\alpha_c = 9.9 \cdot 10^{-6}/\text{C}$  ?



The aluminum rod AD is fitted with a jacket which is used to apply a hydrostatic pressure of 40 MPa to the 300 mm portion BC of the rod. What is a) the change in the total length AD of the rod, b) the change in the diameter at the middle of the rod if the  $E = 70 \text{ GPa}$  and  $\nu = 0.36$ . Also determine the forces which should be applied to the ends A and D of the rod c) if the axial strain in portion BC is to remain zero as the hydrostatic pressure is applied and d) if the total length AD of the rod is to remain unchanged.



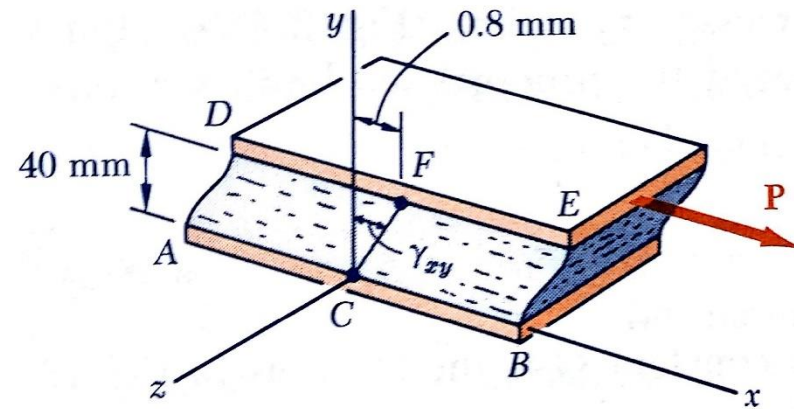
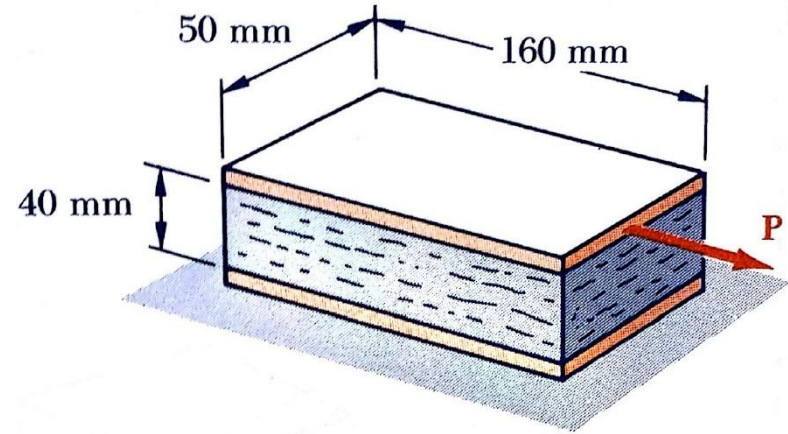
The assembly shown consists of an aluminum shell that is fully bonded to a steel core and is unstressed at a temperature of 20. Compressive centric forces of 180 kN are applied at both ends of the assembly by means of rigid end plates. What are a) the normal stresses in the steel core and the aluminum shell and b) the deformation of the assembly? The moduli of aluminum and steel are  $E_a = 70$  GPa,  $E_s = 200$  GPa.



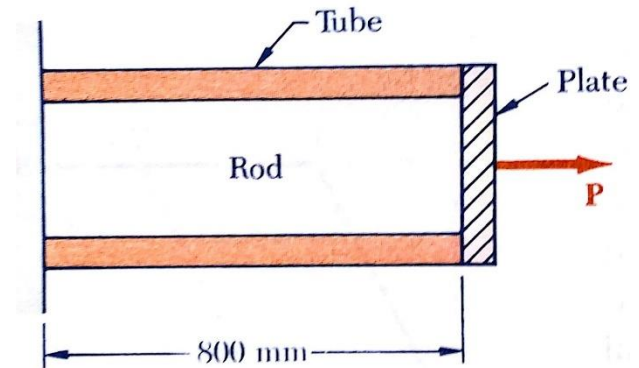
Considering only axial deformations, determine the stress in the aluminum shell when the temperature reaches 180 C. The coefficients of thermal expansion are  $\alpha_a = 23.6 \cdot 10^{-6}/\text{C}$  and  $\alpha_s = 11.7 \cdot 10^{-6}/\text{C}$ .

What is the stress in the aluminum shell if the core is made of brass with  $E_b = 105$  GPa,  $\alpha_b = 20.9 \cdot 10^{-6}/\text{C}$ ?

A rectangular block of a material with a modulus of rigidity (shear modulus)  $G = 600 \text{ MPa}$  is bonded to two rigid horizontal plates. The lower plate is fixed, while the upper plate is subjected to a horizontal force  $P$ . What are the a) average strain in the material and b) the force  $P$  on the upper plate if the upper plate moves through  $0.8 \text{ mm}$  under the action of the force?



An 800 mm long cylindrical rod of cross-sectional area  $A_r = 45 \text{ mm}^2$  is placed inside a tube of the same length and of cross-sectional area  $A_t = 60 \text{ mm}^2$ . The ends of the rod and tube are attached to a rigid support on one side, and to a rigid plate on the other, as shown in the figure. The rod and tube are both assumed to be elastoplastic with moduli of elasticity  $E_r = 200 \text{ GPa}$  and  $E_t = 100 \text{ GPa}$  and yield strengths  $(\sigma_r)_Y = 200 \text{ MPa}$  and  $(\sigma_t)_Y = 250 \text{ MPa}$ . A) What is the load-deflection diagram of the rod-tube assembly when a load  $P$  is applied to the plate. B) What is the elongation of the assembly if the load  $P$  applied is increased from zero to 19.5 kN and decreased back to zero. C) What is the permanent set after the load has been removed.

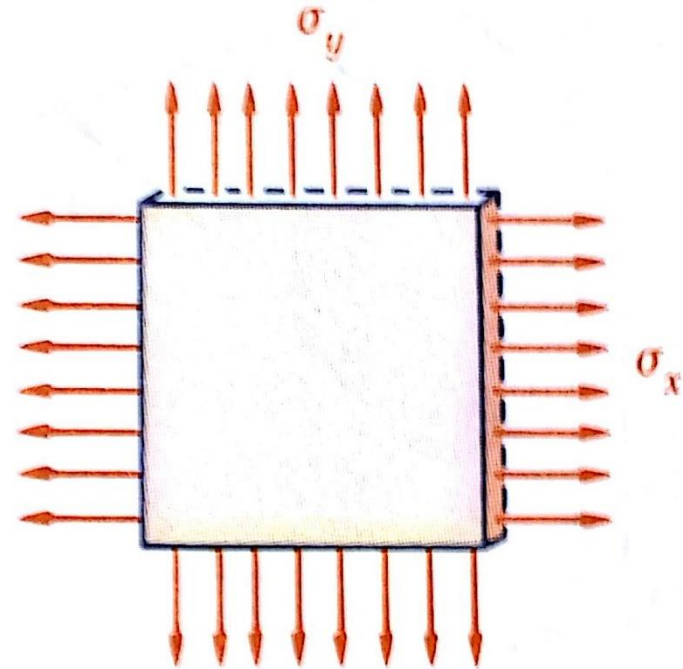


In many situations it is known that the normal stress in a given direction is zero in the case of a thin plate that is under plane stress condition. Show that the following expressions for stresses and strain are correct for the strains in the x and y directions that are determined experimentally.

$$\sigma_x = E \frac{\epsilon_x + \nu \epsilon_y}{1 - \nu^2}$$

$$\sigma_y = E \frac{\epsilon_y + \nu \epsilon_x}{1 - \nu^2}$$

$$\epsilon_z = -\frac{\nu}{1 - \nu} (\epsilon_x + \epsilon_y)$$



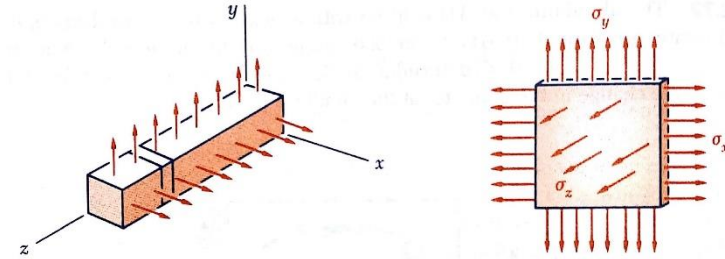


In many situations physical constraints prevent strain from occurring in a given direction. For example  $\epsilon_z = 0$  in the case of plane strain condition where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation the stresses and strains are expressed as:

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\epsilon_x = \frac{1}{E} [(1 - \nu^2)\sigma_x - \nu(1 + \nu)\sigma_y]$$

$$\epsilon_y = \frac{1}{E} [(1 - \nu^2)\sigma_y - \nu(1 + \nu)\sigma_x]$$



A 20 mm square was scribed on the side of a large steel pressure vessel. The plane stress condition of the material after pressurization is shown. What is a) the change in the lengths of the sides and the percent change in the slope of the diagonal DB if  $E = 200 \text{ GPa}$  and  $\nu = 0.3$  ?

A circle of diameter  $d= 200$  mm is scribed on an unstressed aluminum plate of thickness  $t= 18$  mm. Forces acting in the plane of the plate later cause normal stresses  $\sigma_x= 85$  MPa and  $\sigma_z= 150$  MPa. What are the changes in a) the length of diameter AB, b) the length of diameter CD, c) the thickness of the plate, d) the volume of the plate if  $E= 70$  GPa and  $\nu=1/3$ ?

